



# Analytical IPR Equation for Perforated Wells

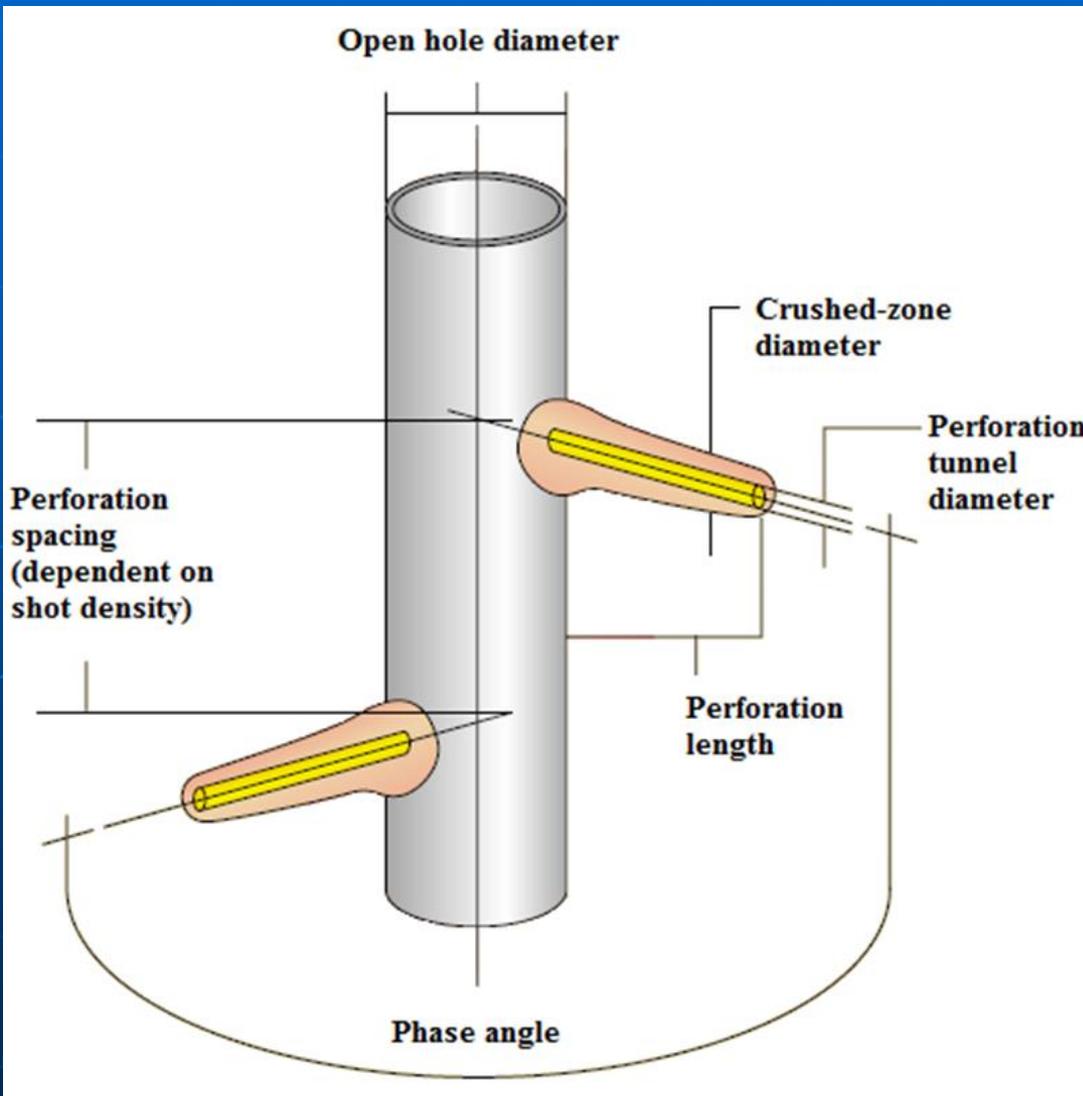
Ádám Pásztor  
Vera Schultz

Petroleum Engineering Department  
University of Miskolc

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**Society of Petroleum Engineers**

# Perforation parameters



Open hole radius	$r_w$
Perforation tunnel radius	$r_p$
Crushed zone radius	$r_c$
Perforation length	$L_p$
Phase angle	$\Theta$
Shot density	$ns$
$k_{crushed}/k_{res}$	$a$

# Main questions of the perforation design



- Which perforation parameter has the most importance?
  - Volume of explosive is limited
  - $L_p$ ,  $r_w$  and  $n_s$  are not independent from each other
- Which phase angle ( $\Theta$ ) to chose?

# Methods for pressure drop calculation

## Method by McLeod:

(McLeod O.H. Jr. 1983)

- Assumes that perforations are small wells and uses the Jones method for pressure drop calculation

## Method by Karakas and Tariq:

(Karakas M. & Tariq S.M. 1988)

- Semi-analytical solution for perforation skin calculation

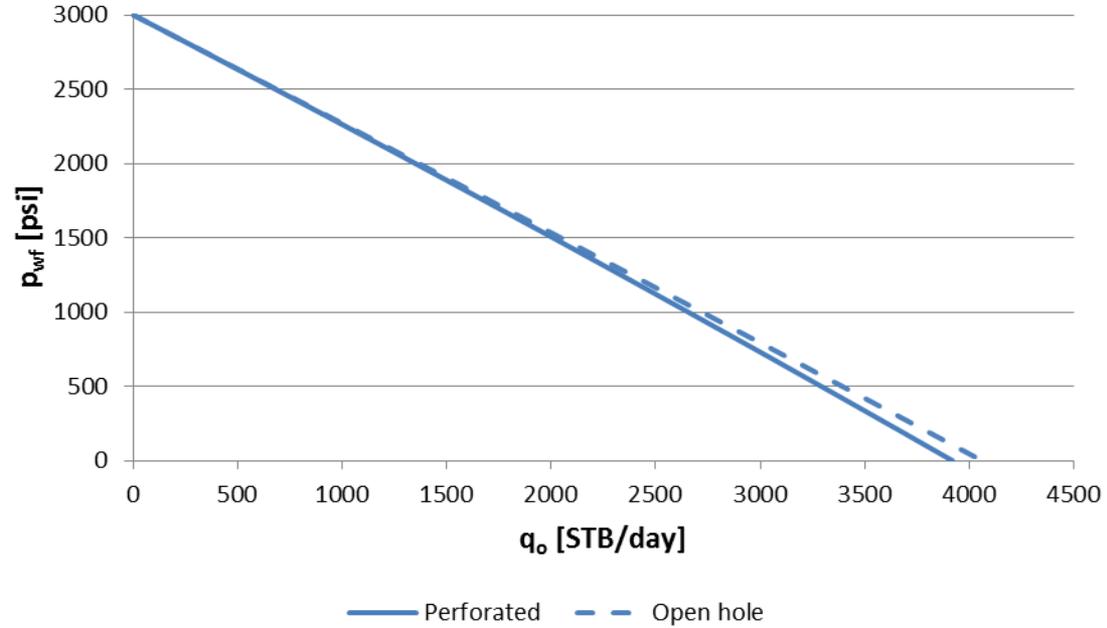
**Investigation with theoretical wells!**

# Data of theoretical wells

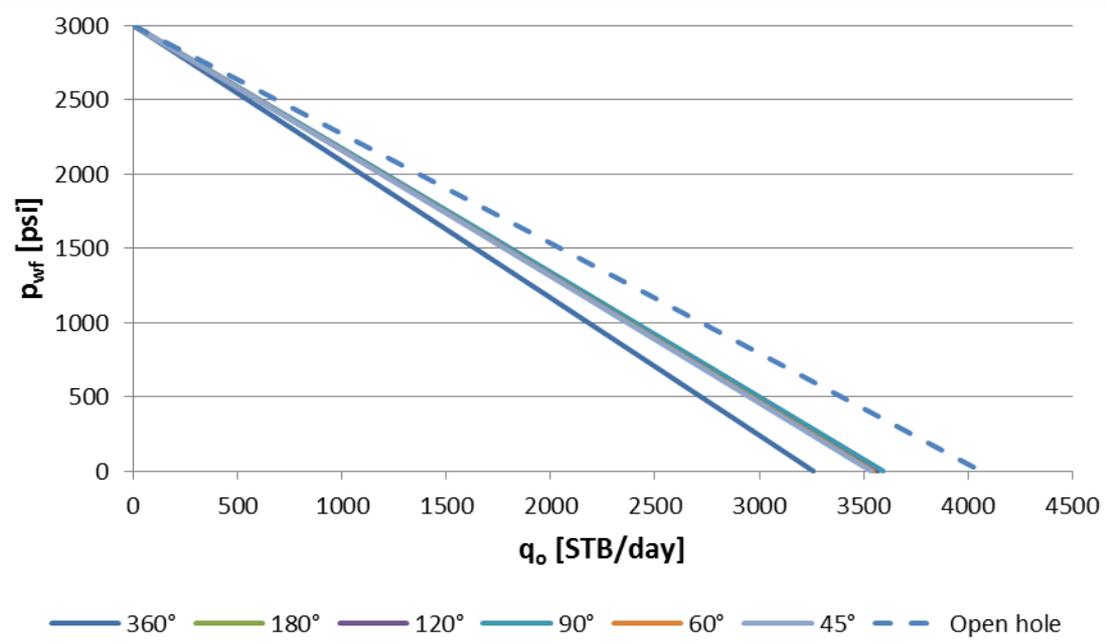
<b><u>Perforation parameters</u></b>						
$L_p$ [ft]	ns [spf]	$\alpha$	$h_p$ [ft]	$\theta$ [°]	$r_p$ [ft]	$r_c$ [ft]
1	5	0.3	25	0	0.015	0.056667
<b><u>Reservoir parameters</u></b>					<b><u>Well parameters</u></b>	
k [mD]	$r_e$ [ft]	$P_r$ [psi]	$k_H$ [mD]	$k_V$ [mD]	$r_w$ [ft]	h [ft]
50	1000	3000	50	5	0.292	25
<b><u>Oil Properties</u></b>			<b><u>Gas Properties</u></b>			
API density	$\mu_o$ [cP]	$B_o$ [bb/STB]	T [R°]	z	$\mu_g$ [cP]	$Y_g$
45.375	0.751	1.16	630	1	0.01933	0.64

# Oil production

McLeod:

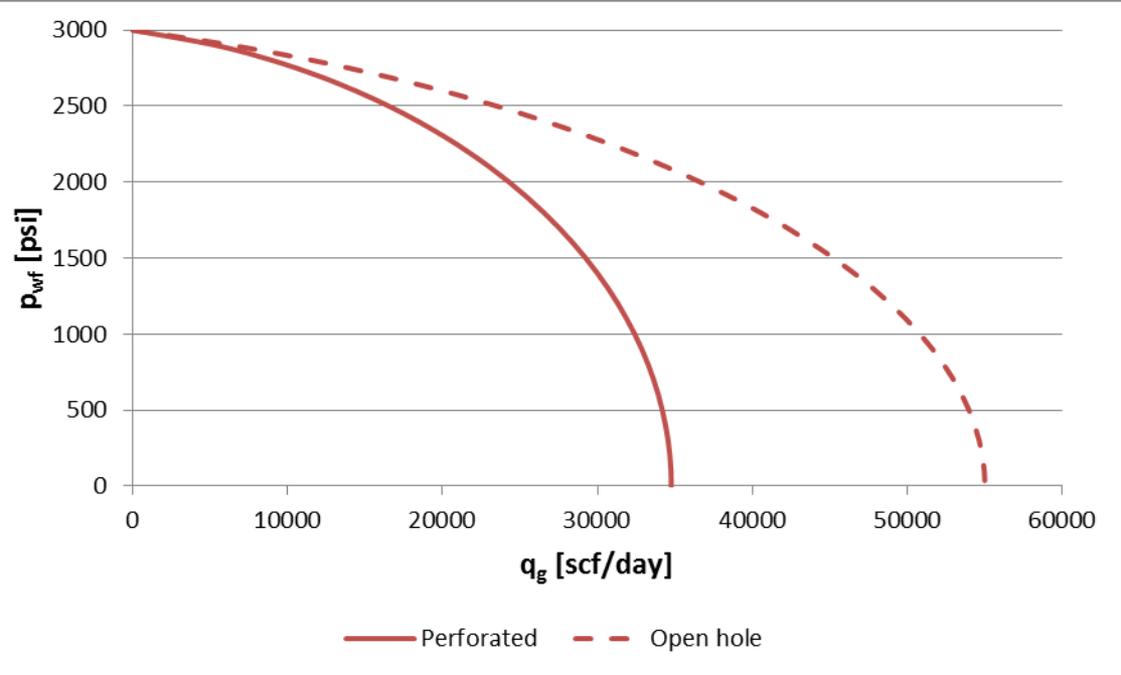


Karakas & Tariq :

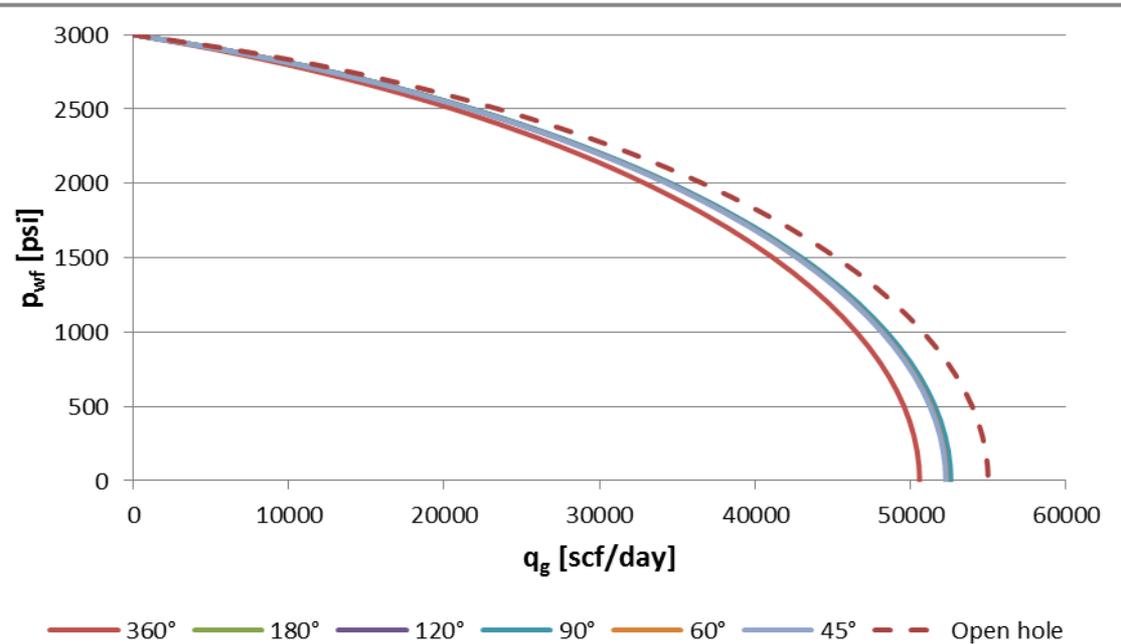


# Gas production

McLeod:



Karakas & Tariq :



## **Conclusion of the investigation:**

- The method of McLeod does not take the phase angle into consideration
- According to the method of Karakas and Tariq:
  - The perforation design has no effect on the non-Darcy term
  - The best phase angle is  $90^\circ$  (not explained)

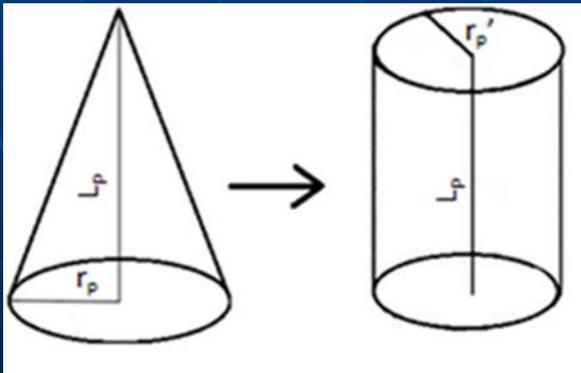
## **Criteria for a new IPR equation:**

- It should have a purely analytical derivation.
- The phase angle must be taken into consideration.
- It must modify both the non-Darcy and Darcy terms.

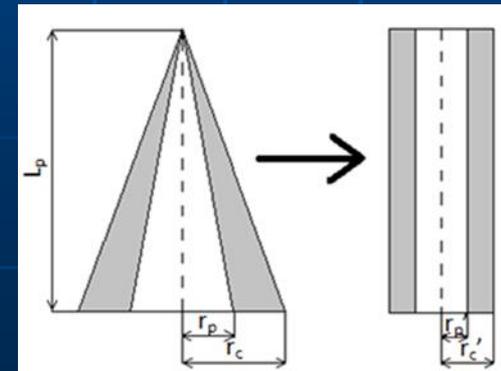
# Analytical IPR equation – Base concept

- The flow is separated into two sections:
  - Flow perpendicular to the axis of the well
  - Flow perpendicular to the axis of the perforation channels
- The perforations are assumed to be small wells .
- Modification of the radius of the perforation channels and the crushed zone (Pásztor Á. & Kosztin B. 2015).

## Modification of $r_p$ :

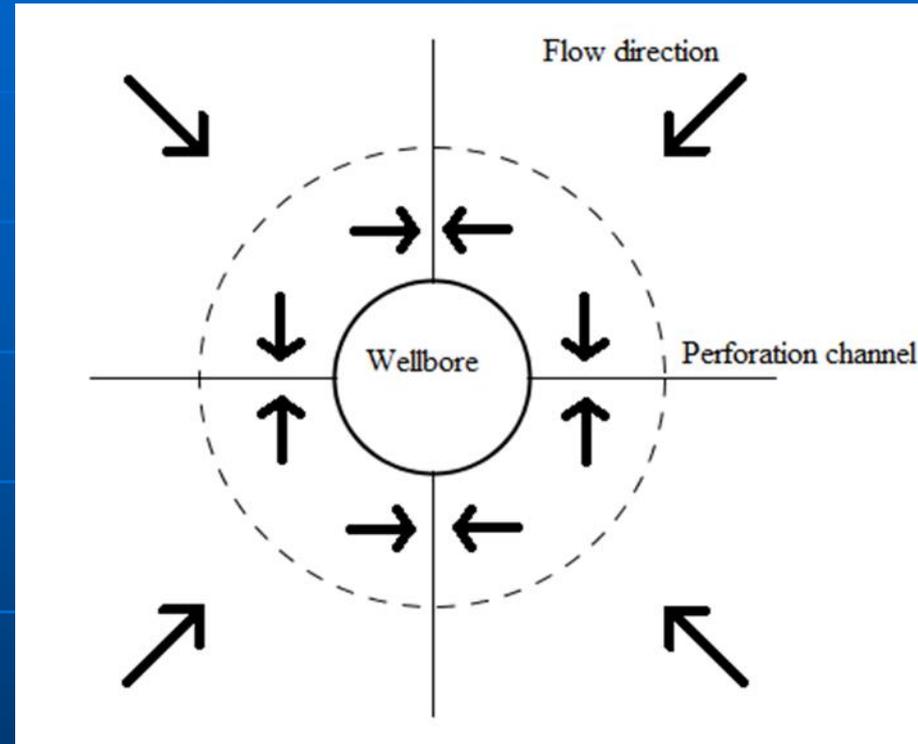


## Modification of $r_c$ :

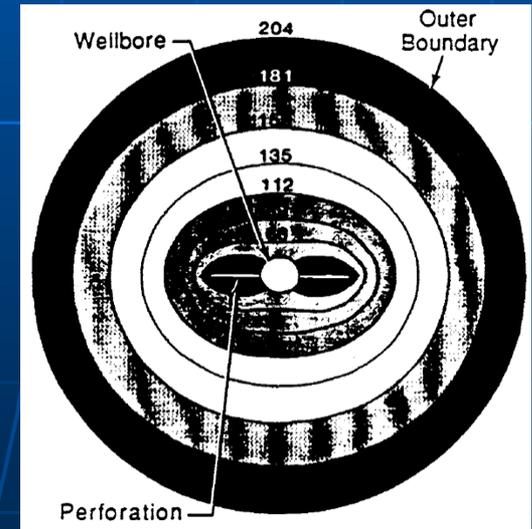
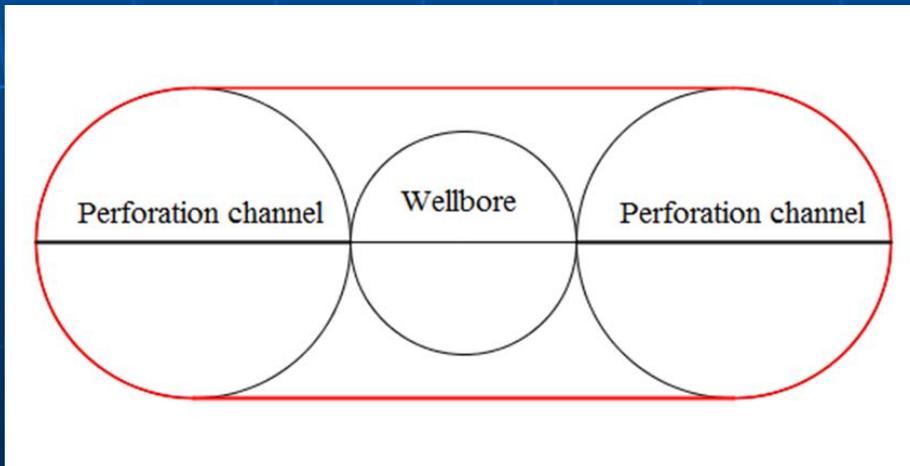
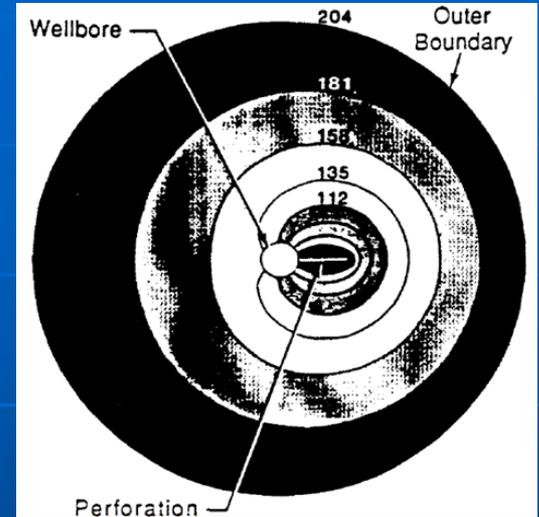
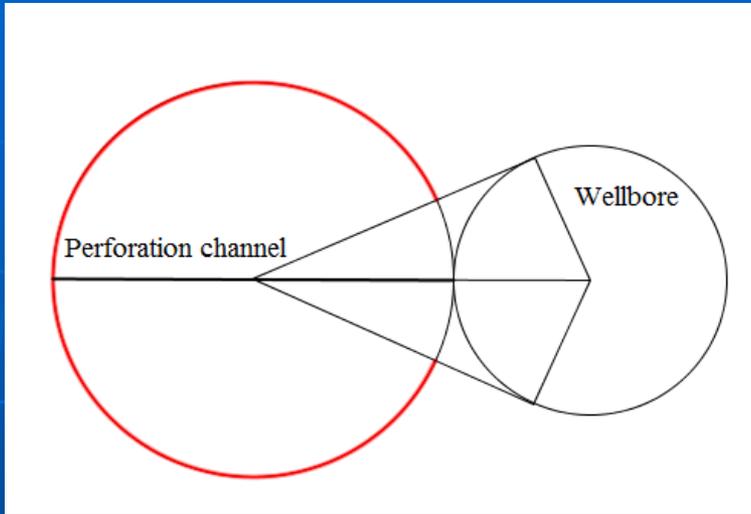


# Extended wellbore radius

- The distance from the axis of the well at which the flow changes direction can be assumed as the radius of an extended wellbore.
- The flow direction of an average particle changes at the distance from the axis of the well where the volume of the drainage area is halved.



# Extended wellbore radius for $\theta=360^\circ$ , $180^\circ$



(After Karakas M. & Tariq S.M. 1988)

# IPR of a perforation channel

$$A = C_1 \times \frac{1}{h_p^2} \left( \frac{1}{r_w} - \frac{1}{r_e} \right) \longrightarrow A_p = C_1 \times \frac{\frac{1}{r_p} - \frac{1}{r_{ep}}}{L_p^2}$$

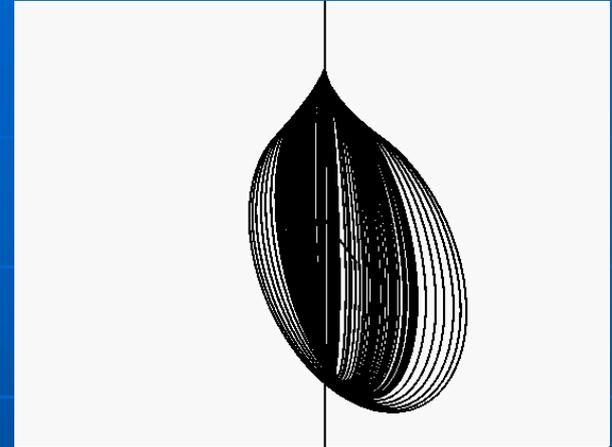
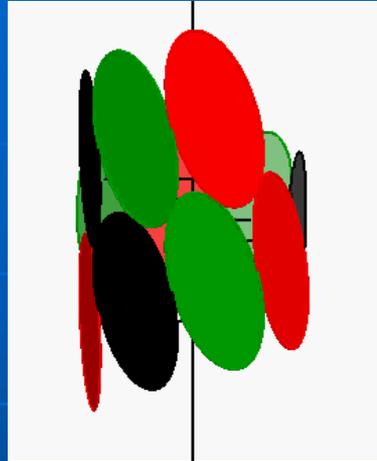
$$B = C_2 \times \frac{\ln \left( 0.472 \left( \frac{r_e}{r_w} \right) \right) + S}{h} \longrightarrow B_p = C_2 \times \frac{\ln \left( 0.472 \frac{r_{ep}}{r_p} \times \left( \frac{r_c}{r_p} \right)^{\frac{1-\alpha}{\alpha}} \right)}{L_p}$$

A and B parameters from the IPR equation of Jones et al. (Jones L.G. et al. 1967)

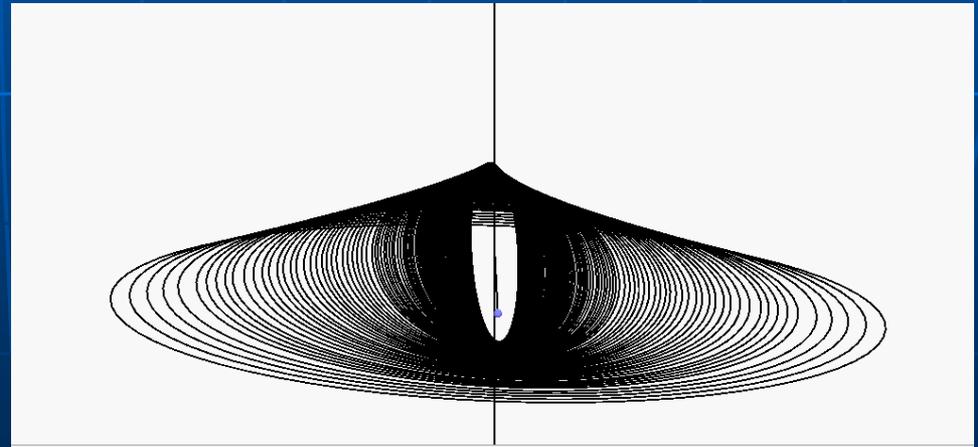
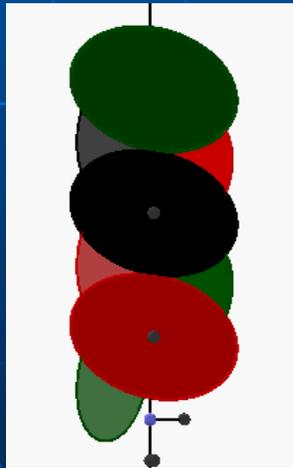
$r_{ep}$  is the radius of a cylinder which has the same length as the perforations and the same area as the perforation channel's drainage space

# Shape of the perforation channels' drainage space

$\Theta = 45^\circ$



$\Theta = 120^\circ$



# Final form of the analytical IPR equation

$$p_r^{(2)} - p_{wf}^{(2)} = q^2 A + qB$$

$$A = C_1 \times \frac{1}{h_p^2} \left( \frac{1}{r_w} \right) (\lambda_{ewb} + \lambda_p)$$

$$B = C_2 \times \frac{\ln \left( 0.472 \left( \frac{r_e}{r_w} \right) \right) + S_{ewb} + S_p}{h}$$

For oil production:

$$C_1 = 5.359 \times 10^{-4} \frac{B_o^2 \rho}{k^{1.201}}$$

$$C_2 = 141.24 \times \frac{\mu_o B_o}{k}$$

$$S_p = \frac{\ln \left( 0.472 \frac{r_{ep}}{r'_{pe}} \times \left( \frac{r'_{ce}}{r'_{pe}} \right)^{\frac{1-\alpha}{\alpha}} \right)}{L_p n s h_p} \times h$$

$$S_{ewb} = \ln \left( \frac{r_w}{r_{ewb}} \right)$$

$$\lambda_p = \frac{\frac{r_w}{r'_{pe}} - \frac{r_w}{r_{ep}}}{L_p^2 n s^2} \quad \lambda_{ewb} = \frac{r_w}{r_{ewb}}$$

For gas production:

$$C_1 = 7.3628 \times 10^{-2} \frac{\gamma_g T z}{k^{1.201}}$$

$$C_2 = 1424 \times \frac{\mu_g T z}{h}$$

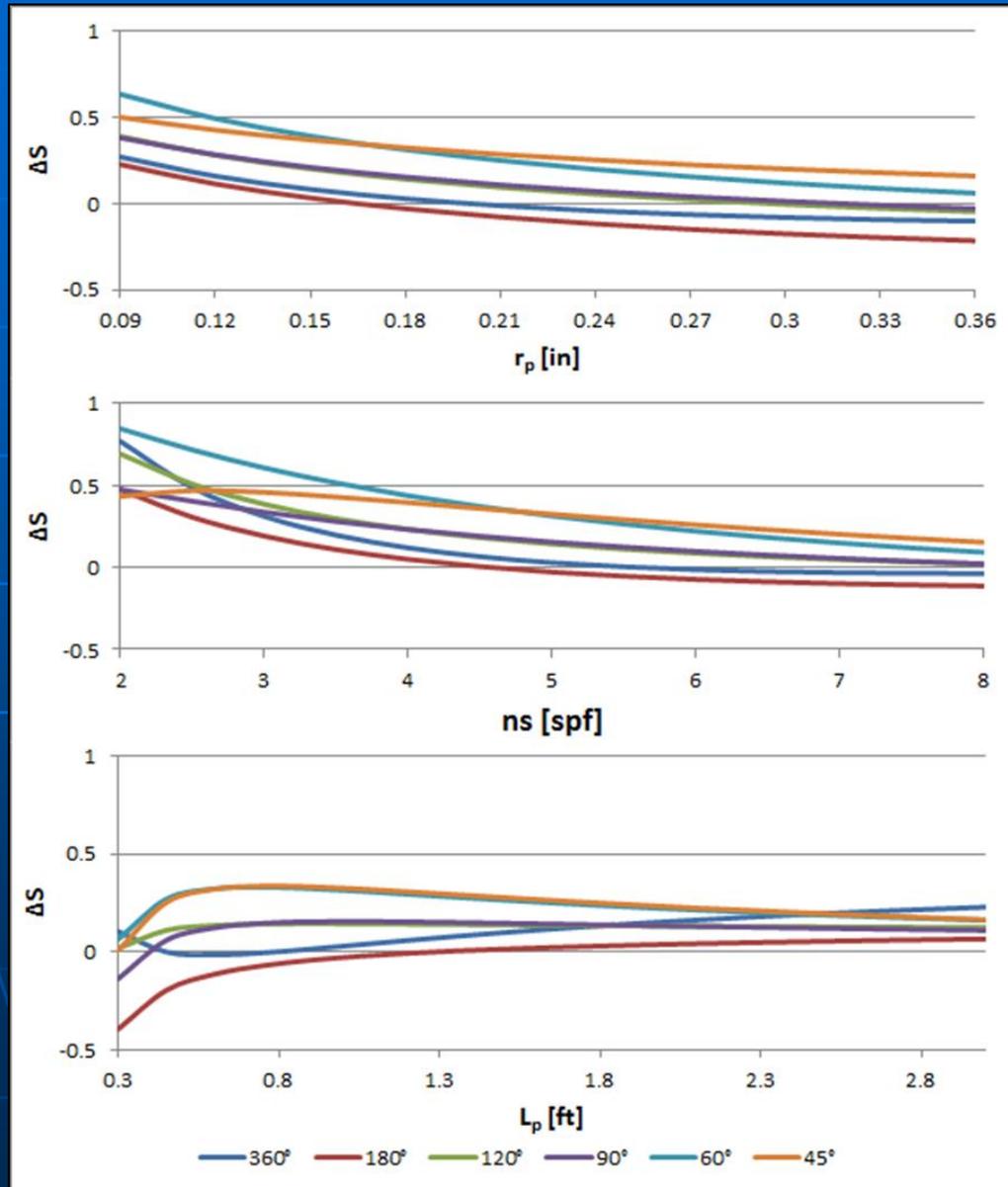
# Analysis of the analytical equation's behavior

- Comparison of the rate independent skin factors
- IPR curves of the theoretical wells
- Impact of perforation parameters on the productivity

Parameters of the sensitivity tests:

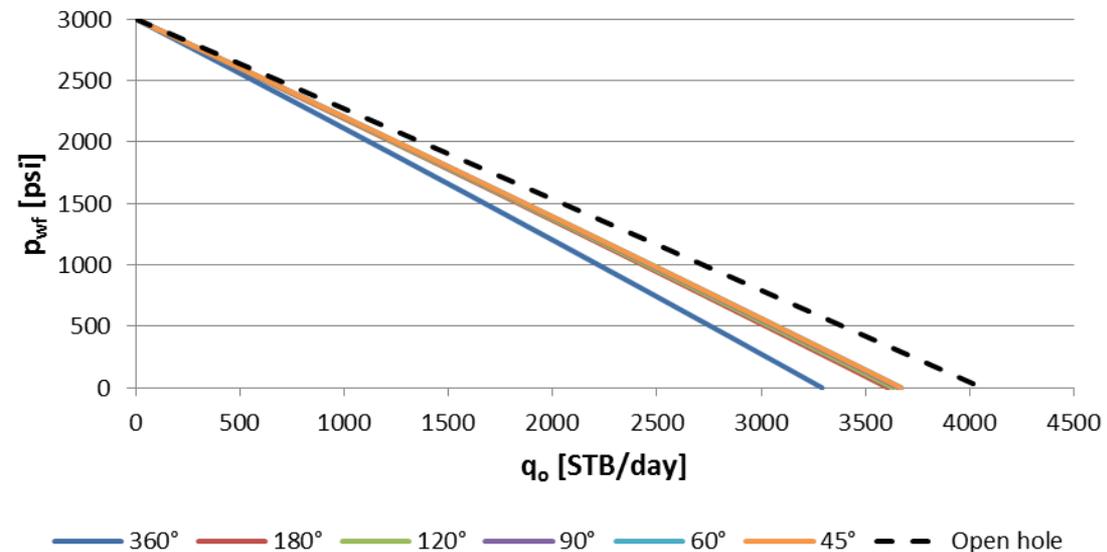
<b><u>Variable</u></b>	<b><u>Starting value</u></b>	<b><u>End value</u></b>
<b>Shot density (<math>n_s</math>) [spf]</b>	2	8
<b>Perforation length (<math>L_p</math>) [ft]</b>	0.3	3
<b>Perforation channel radius (<math>r_p</math>) [in]</b>	0.09	0.36

# Comparison of the rate independent skin factors

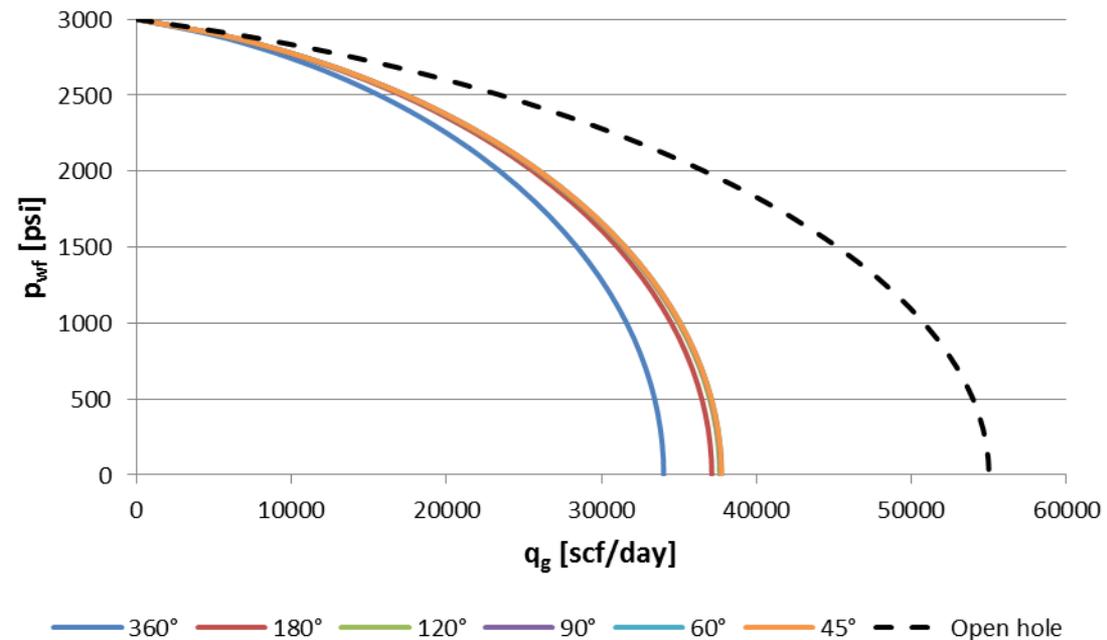


# IPR curves of the theoretical wells

Oil production:

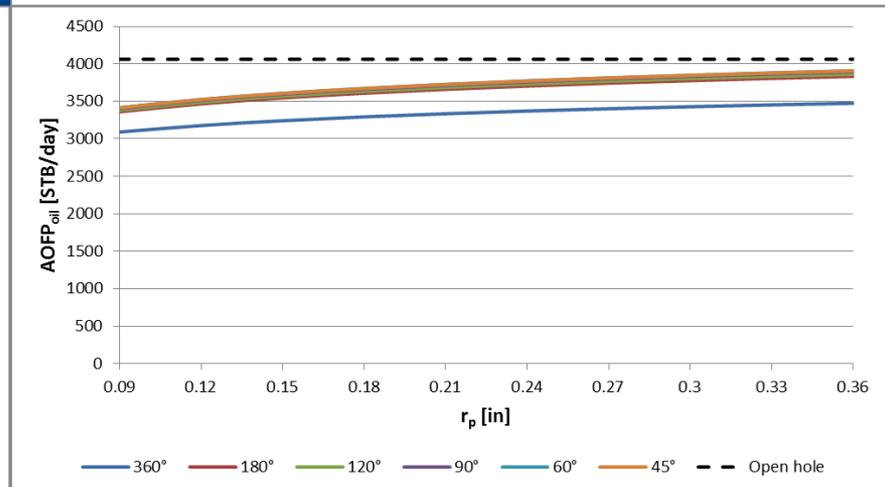
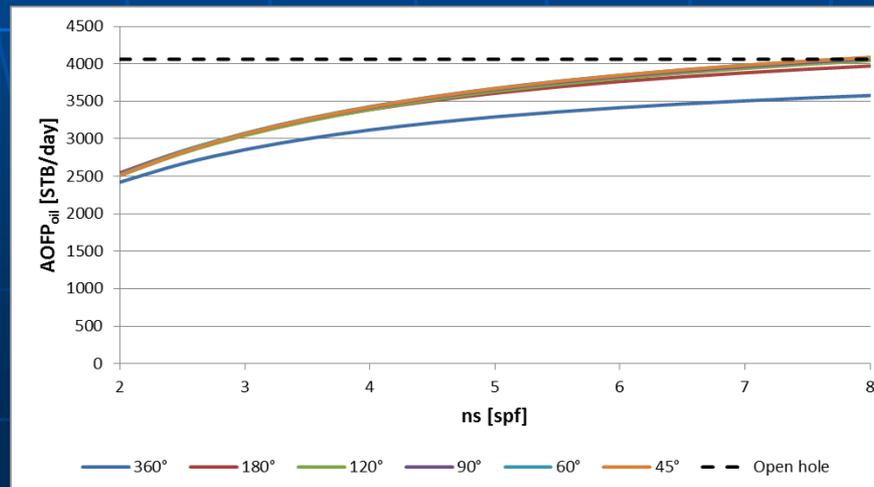
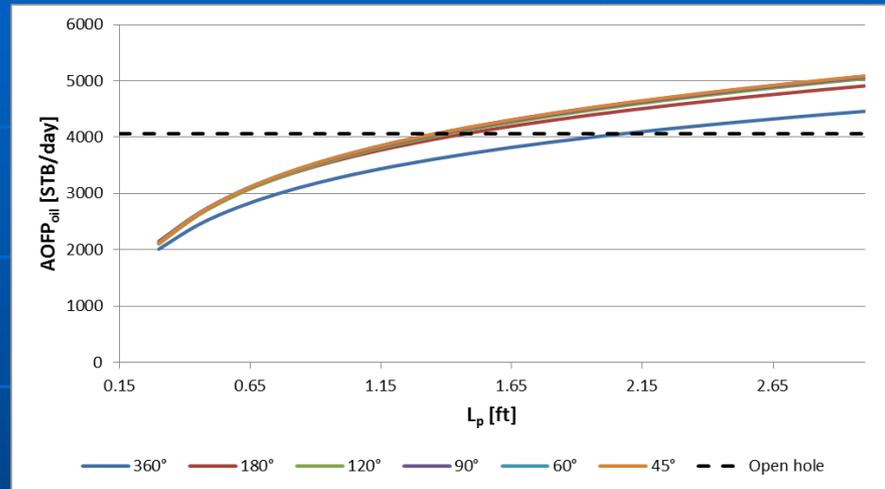


Gas production:



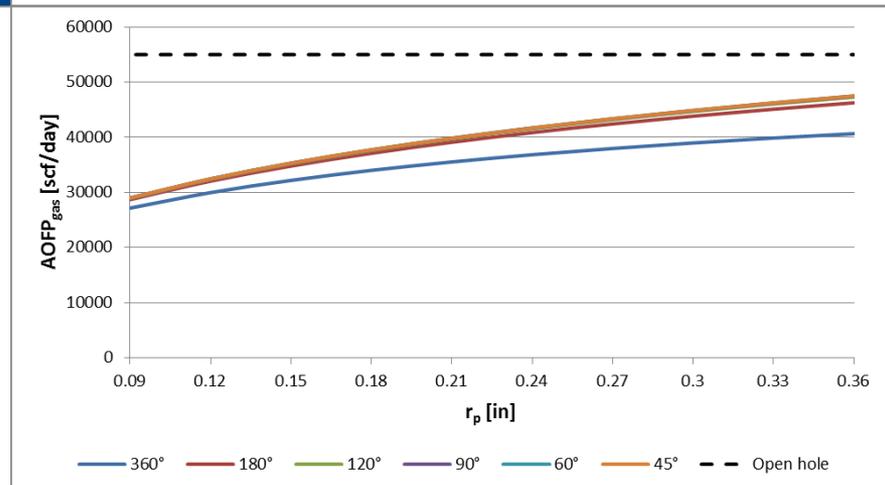
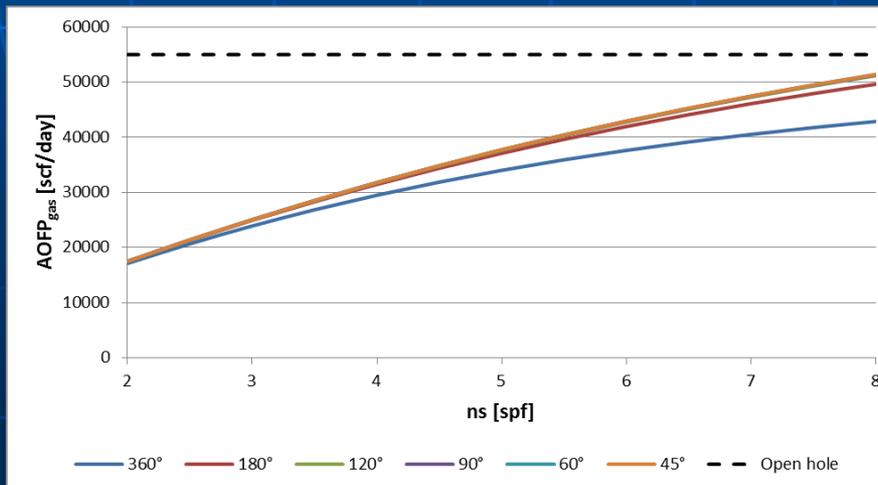
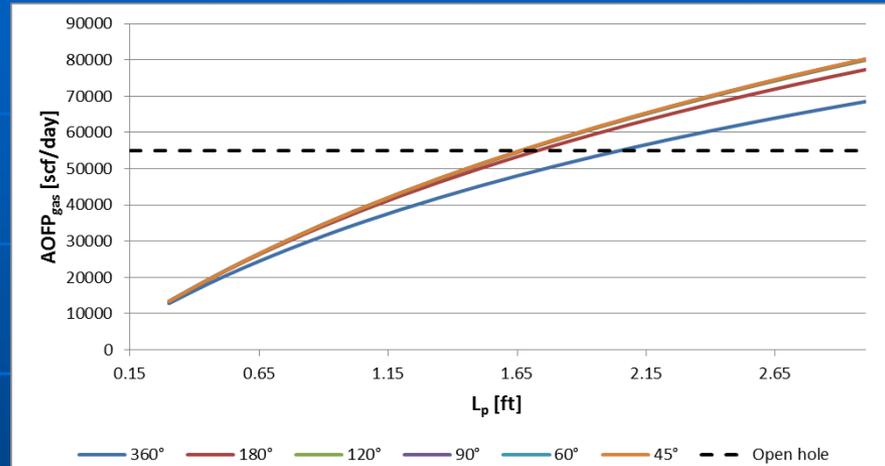
# Impact of perforation parameters on the productivity

## Oil production



# Impact of perforation parameters on the productivity

## Gas production



# Conclusion

- All the previously set criteria are met.
- The analytical equation describes the results of Karakas and Tariq well.
- The best perforation angle is  $45^\circ$ .
- The perforation channel length has the greatest effect on the productivity and the perforation channel radius has the smallest.
- With a proper perforation design the productivity of a perforated well can be better than a well with an open hole completion.
- In case of gas production it is more difficult to achieve a better productivity than in the case of open hole completed wells due to the rate dependent skin.

# References

- Jones, L.G., Blount, E.M., and Glaze, O.H.: "Use of Short- Term Multiple-Rate Flow Test to Predict Performance of Wells Having Turbulence," paper SPE6133 presented at the 1967 SPE Annual Technical Conference and Exhibition, New Orleans, Oct. 3-6.
- McLeod, O.H. Jr.: "The Effect of Perforating Conditions on Well Performance," JPT (Jan. 1983) 31-39.
- Karakas, M., and Tariq, S. M.: "Semi-analytical Productivity Models for Models for Perforated Completions," paper SPE 18247 presented at the 63rd Annual Technical Conference and Exhibition of the SPE held in Houston, TX, October 2-5, 1988.
- Pásztor, Á., & Kosztin, B. (2015, June 3). A Novel Method for Optimal Perforation Design. Society of Petroleum Engineers. doi:10.2118/174207-MS

**Thank you for your kind attention!**  
**Questions?**